

PRAARAS

JEE 2026

Mathematics

Basic Maths

Lecture - 06

By – Ashish Agarwal Sir
(IIT Kanpur)



Topics *to be covered*



- A** Problem Practice
- B** Algebraic Identities
- C** Laws of Exponent



Homework Discussion

QUESTION



If $81x^5 + 27x^3 - 9x^2 + 50$ is divided by $(3x + 2)$ then remainder is λ then

- A** $\frac{3\lambda - 42}{10}$ is equal to 4
- B** if $\lambda = \frac{p}{q}$ then $(p + q)$ is divisible by 17 (where p & q are coprime)
- C** λ is a natural number
- D** $\left(\lambda - \frac{1}{3}\right)$ is divisible by 3

Ans. A, B, D

Let $f(x) = x^{135} + x^{125} - x^{115} + x^5 + 1$. If $f(x)$ is divided by $x^3 - x$ then the remainder is some function of x say $g(x)$. Find the value of $g(10)$.

$$f(x) = (x^3 - x) \cdot Q(x) + ax^2 + bx + c$$

$$g(x) = ax^2 + bx + c$$

$$f(x) = x(x-1)(x+1) \cdot Q(x) + ax^2 + bx + c$$

Put $x=1$, $f(1) = a + b + c$
 $f(0) = c$
 $f(-1) = a - b + c$

$\left. \begin{array}{l} a + b + c \\ c \\ a - b + c \end{array} \right\} \begin{array}{l} a = 0 \\ b = 2 \\ c = 1 \end{array}$

If $p(x)$ is a polynomial of 3 degree for which $p(1) = 1$, $p(2) = 4$, $p(3) = 9$ then find the value of $p(5)$ and leading coefficient be 2.

$$\text{degree 3} \quad g(x) = p(x) - x^2$$
$$g(1) = g(2) = g(3) = 0$$

$$g(x) = 2(x-1)(x-2)(x-3)$$

$$p(x) - x^2 = 2(x-1)(x-2)(x-3)$$

$$p(x) = 2(x-1)(x-2)(x-3) + x^2$$

**Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal**

QUESTION



Three real numbers x, y, z are such that $x^2 + 6y = -17$, $y^2 + 4z = 1$ and $z^2 + 2x = 2$. Then the value of $x^3 + y^3 + z^3$ is equal to

- A** 30
- B** -24
- C** -36
- D** -28

$$y^2 + 2 \cdot 3 \cdot y$$

$$x^2 + y^2 + z^2 + 6y + 4z + 2x = -14$$

$$x^2 + 2x + 1 + y^2 + 6y + 9 + z^2 + 4z + 4 = -14 + 14$$

$$(x+1)^2 + (y+3)^2 + (z+2)^2 = 0$$

$$x = -1, y = -3, z = -2$$

All 3 satisfied

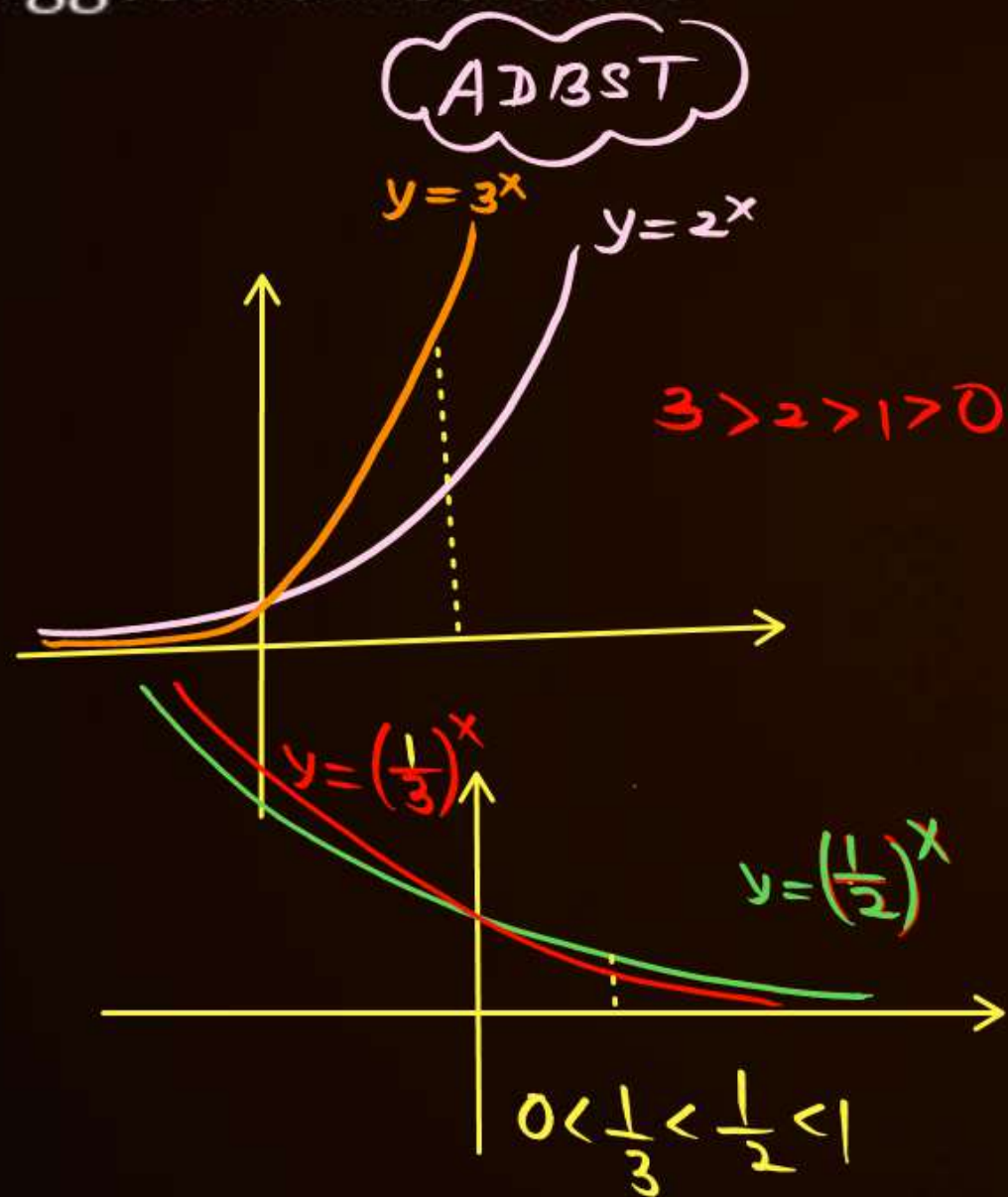
$$x^3 + y^3 + z^3 = -1 - 27 - 8 = -36$$

QUESTION



Suppose that $w = 2^{1/2}$, $x = 3^{1/3}$, $y = 6^{1/6}$ and $z = 8^{1/8}$. From among these number list, the biggest, second biggest numbers are

- A** w, x
- B** x, w
- C** y, z
- D** x, z



$$* 0 < a < 1 \quad a^x \downarrow \text{ as } x \uparrow$$

$$* a > 1 \quad a^x \uparrow \text{ as } x \uparrow$$

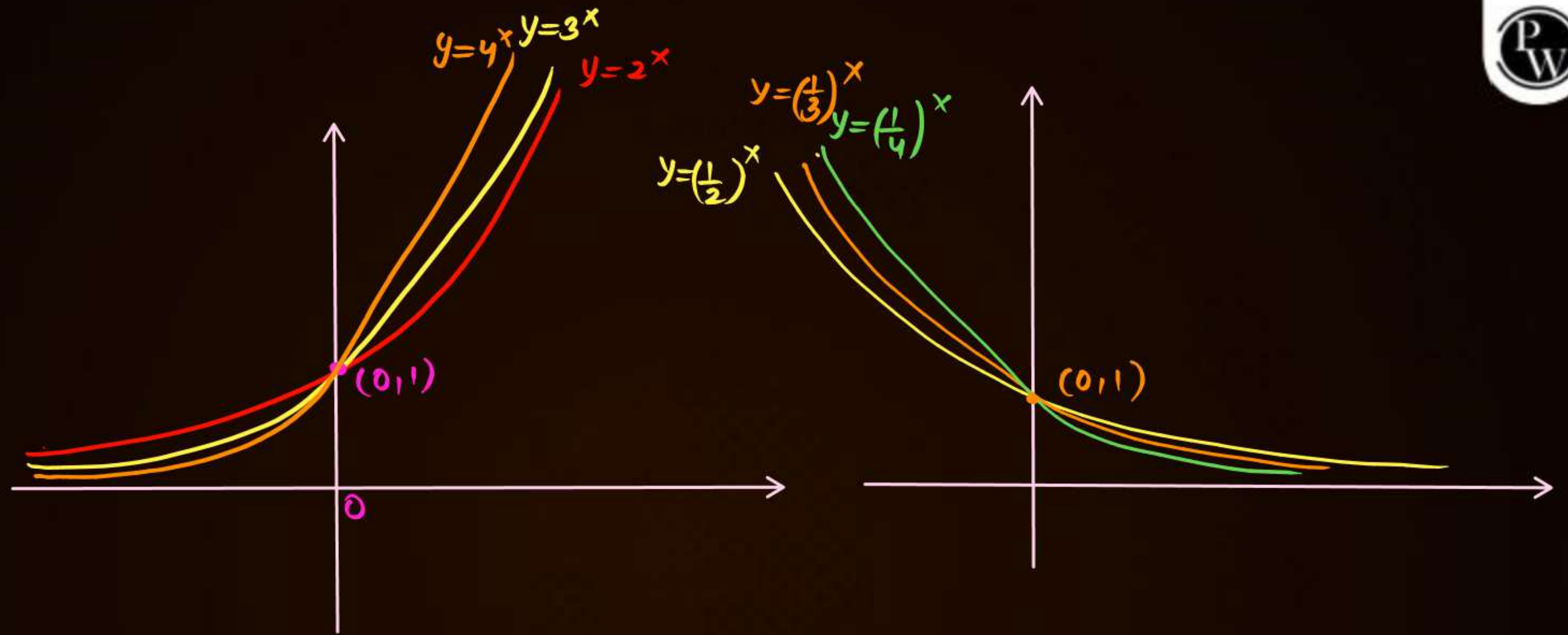
$$\textcircled{1} a > b > 0 \Rightarrow a^x > b^x \quad x > 0 - (\text{True})$$

$$\textcircled{2} a > b > 1 \Rightarrow a^{-x} < b^{-x} \quad x > 0 - \text{True}$$

$$\textcircled{3} 0 < b < a < 1 \Rightarrow a^{-x} < b^{-x}, x > 0 - \text{True}$$

$\underbrace{\frac{1}{3}}_{\text{-ve power}} \quad \underbrace{\frac{1}{2}}_{\text{-ve power}}$

$$\textcircled{4} 0 < b < a < 1 \Rightarrow a^x > b^x \quad (x > 0; \text{True})$$



QUESTION

$$\text{lcm}(1, 2, 3) = 6$$

$$6\left(\frac{\lambda}{6}\right) = 6\mu$$



If a , b & c are three non zero real numbers such that $4a^2 + 9b^2 + c^2 - 6ab - 3bc - 2ca = 0$ then the value of $(4a + c)/b$ is equal to

$$4a^2 + 9b^2 + c^2 - 6ab - 3bc - 2ca = 0$$

$$(2a)^2 + (3b)^2 + c^2 - 2a \cdot 3b - 3b \cdot c - c \cdot 2a = 0$$

$$2a = 3b = c = 6\lambda$$

$$\begin{matrix} a = 3\lambda \\ b = 2\lambda \\ c = 6\lambda \end{matrix} \Rightarrow \frac{4a + c}{b} = \frac{12\lambda + 6\lambda}{2\lambda} = 9$$

QUESTION



If $a, b, c \in \mathbb{R}$ then find the minimum value of $E = a^2 + 9b^2 + 25c^2 + 2a + 6b - 10c + 20$.

$$E = a^2 + 2a + 9b^2 + 6b + 25c^2 - 10c + 20$$

$$E = a^2 + 2a + 1^2 + (3b)^2 + 2 \cdot 3b \cdot 1 + 1^2 + (5c)^2 - 2 \cdot 5c \cdot 1 + 1^2 + 20 - 3$$

$$= \underbrace{(a+1)^2}_{\geq 0} + \underbrace{(3b+1)^2}_{\geq 0} + \underbrace{(5c-1)^2}_{\geq 0} + 17$$

$$E_{\min} = 17 \quad \text{at } a = -1$$

$$b = -1/3$$

$$c = 1/5$$

QUESTION

Tahoi



The number of real number pairs (x, y) which will satisfy the equation $x^2 - xy + y^2 = 4(x + y - 4)$ is

Ans. 1

QUESTION



If α, β and γ are roots of cubic equation $x^3 + 3x - 1 = 0$ then find value of:

- ~~(i)~~ $(2 - \alpha)(2 - \beta)(2 - \gamma)$
- ~~(ii)~~ $(3 + \alpha)(3 + \beta)(3 + \gamma)$
- (iii) $(4 - \alpha^2)(4 - \beta^2)(4 - \gamma^2)$
- ~~(iv)~~ $(1 + \alpha^2)(1 + \beta^2)(1 + \gamma^2)$

$$x^3 + 3x - 1 \leftarrow \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix}$$

$$x^3 + 3x - 1 = (x - \alpha)(x - \beta)(x - \gamma)$$

for (i) put $x = 2$

for (ii) put $x = -3$

$$-27 - 9 - 1 = (-3 - \alpha)(-3 - \beta)(-3 - \gamma)$$

$$-37 = -(3 + \alpha) \cdot -(3 + \beta) \cdot -(3 + \gamma)$$

$$-37 = -(3 + \alpha)(3 + \beta)(3 + \gamma)$$

$$(3 + \alpha)(3 + \beta)(3 + \gamma) = 37$$

for (iii)

put $x = 2$

$$(2 - \alpha)(2 - \beta)(2 - \gamma) = 13 \quad \text{--- (i)}$$

put $x = -2$

$$(-2 - \alpha)(-2 - \beta)(-2 - \gamma) = -15$$

$$(2 + \alpha)(2 + \beta)(2 + \gamma) = 15 \quad \text{--- (ii)}$$

$$(4 - \alpha^2)(4 - \beta^2)(4 - \gamma^2) = 195 \text{ Ans.}$$

$$* x^2 + y^2 = x^2 - (iy)^2 = (x+iy)(x-iy)$$

$$* 1+a^2 = (1+ia)(1-ia) = (a+i)(a-i)$$

$$i = \sqrt{-1} \Rightarrow i^2 = -1$$

$$i^3 = i \times i \times i = -i$$

$$(-i)^3 = -i \times -i \times -i = -(i \times i \times i) = i$$

$$(2i)^2 = 2i \times 2i = 4 \times i^2 = 4 \times -1 = -4$$

for (iv)

put $x=i$

, put $x=-i$

$$-i \quad \curvearrowright \quad i^3 + 3i - 1 = (i-\alpha)(i-\beta)(i-\gamma) \Rightarrow 2i-1 = (i-\alpha)(i-\beta)(i-\gamma) \quad \text{--- ①}$$

$$+i \quad \curvearrowright \quad (-i)^3 - 3i - 1 = (-i-\alpha)(-i-\beta)(-i-\gamma) \Rightarrow -2i-1 = -(i+\alpha)(i+\beta)(i+\gamma)$$

$$2i+1 = (i+\alpha)(i+\beta)(i+\gamma) \quad \text{--- ②}$$

$$\textcircled{①} \cdot \textcircled{②} \quad (2i)^2 - 1^2 = (i^2 - \alpha^2)(i^2 - \beta^2)(i^2 - \gamma^2)$$

$$-5 = -(1+\alpha^2) \cdot -1 \cdot (1+\beta^2) \cdot -1(1+\gamma^2)$$

$$(1+\alpha^2)(1+\beta^2)(1+\gamma^2) = 5.$$



Factorization of Polynomial

Ex: $P(x) = x^3 - 6x^2 + 11x - 6$ \curvearrowright $P(1) = 1 - 6 + 11 - 6 = 0 \Rightarrow (x-1)$ is a factor

$$\begin{aligned} P(x) &= x^2(x-1) - 5x(x-1) + 6(x-1) \\ &= (x-1)(x^2 - 5x + 6) \\ &= (x-1)(x^2 - 3x - 2x + 6) \\ &= (x-1)(x-3)(x-2) \end{aligned}$$

$$x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$$

$$x^3 + 6x^2 + 11x + 6 = (x+1)(x+2)(x+3)$$

QUESTION



Factorize the following

Tah02

(i) $x^3 - 13x - 12$

[Ans. $(x + 1)(x - 4)(x + 3)$]

(ii) $x^3 - 7x - 6$

[Ans. $(x + 2)(x - 3)(x + 1)$]

(iii) $x^3 - 6x^2 + 11x - 6$

[Ans. $(x - 1)(x - 2)(x - 3)$]

(iv) $2x^3 + 9x^2 + 10x + 3$

[Ans. $(x + 1)(x + 3)(2x + 1)$]

(v) $x^3 - 9x^2 + 23x - 15$

[Ans. $(x - 1)(x - 3)(x - 5)$]

(vi) $2x^3 - 9x^2 + 13x - 6$ $\rightarrow P(1) = 2 - 9 + 13 - 6 = 0$

[Ans. $(x - 1)(x - 2)(2x - 3)$]

(vii) $x^3 - 4x^2 + 5x - 2$

[Ans. $(x - 2)(x - 1)^2$]

$$\begin{aligned} & 2x^2(x-1) - 7x(x-1) + 6(x-1) \\ & (2x^2 - 7x + 6)(x-1) \\ & (2x^2 - 4x - 3x + 6)(x-1) \\ & (2x-3)(x-2)(x-1) \end{aligned}$$



Algebraic Identities

$$I_1: (a \pm b)^2 = a^2 + b^2 \pm 2ab$$

$$I_2: a^2 - b^2 = (a - b)(a + b)$$


$$I_3: a^3 + b^3 = (a + b)(a^2 - ab + b^2) = (a + b)^3 - 3ab(a + b)$$


$$I_4: a^3 - b^3 = (a - b)(a^2 + ab + b^2) = (a - b)^3 + 3ab(a - b)$$

$$I_5: (a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$I_6: (a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$I_7: (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \\ = a^2 + b^2 + c^2 + 2abc(1/a + 1/b + 1/c)$$

$$I_8: (a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)$$


$$I_9: a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$


$a, b, c \in \mathbb{R}$

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

S_1 : If $a+b+c=0$ then $a^3 + b^3 + c^3 = 3abc$ (True / False) ✓

S_2 : If $a^3 + b^3 + c^3 = 3abc$ then $a+b+c=0$ (True / False) ✓

$$a^3 + b^3 + c^3 = 3abc$$

$$a^3 + b^3 + c^3 - 3abc = 0$$

$$(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$$

$$a+b+c=0 \text{ or } a^2 + b^2 + c^2 - ab - bc - ca = 0$$

$$\Downarrow$$

$$a=b=c$$

$$a^3 + b^3 + c^3 = 3abc \Leftrightarrow a+b+c=0 \text{ or } a=b=c.$$



(ASNC)

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$$

$$(a+b+c+d)^2 = a^2 + b^2 + c^2 + d^2 + 2(\underbrace{ab+ac+ad+bc+bd+cd}_{\downarrow \text{sum of products taken two at a time}})$$

sum of products
taken two at a time

$$(a_1+a_2+\dots+a_n)^2 = a_1^2+a_2^2+a_3^2+\dots+a_n^2+2(a_1a_2+a_1a_3+\dots+\dots+a_{n-1}a_n)$$

$$(a-b+c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$$

$$(a+b-c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$$

$$(a-b-c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$$

Kya ye sab use hotaa hai JEE mai ???

If $a + b + c = 1$, $ab + bc + ca = 2$ and $abc = 3$, then the value of $a^4 + b^4 + c^4$ is equal to

S.B.S

$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 1$$

$$a^2 + b^2 + c^2 + 4 = 1$$

$$a^2 + b^2 + c^2 = -3$$

S.B.S

$$a^4 + b^4 + c^4 + 2(a^2b^2 + b^2c^2 + c^2a^2) = 9$$

$$(ab + bc + ca)^2 = 4$$

$$a^2b^2 + b^2c^2 + c^2a^2 + 2(ab^2c + bc^2a + a^2bc) = 4$$

$$a^2b^2 + b^2c^2 + c^2a^2 + 2abc(b + c + a) = 4$$

$$a^2b^2 + b^2c^2 + c^2a^2 + 2 \cdot 3 \cdot 1 = 4 \rightarrow a^2b^2 + b^2c^2 + c^2a^2 = -2$$

$$a^4 + b^4 + c^4 - 4 = 9$$

$$a^4 + b^4 + c^4 = 13$$

QUESTION

Tah03



Given that $a + b + c = 3$, $a^2 + b^2 + c^2 = 5$ and $a^3 + b^3 + c^3 = 7$, then the value of $a^4 + b^4 + c^4$ is equal to

QUESTION



If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$ then $(a + b + c)^3 =$

A abc

B $3abc$ — 37.1.

C $9ac$

~~**D**~~ $27abc$ — 43.1.

$$\rightarrow (\sqrt[3]{a})^3 + (\sqrt[3]{b})^3 + (\sqrt[3]{c})^3 = 3 \sqrt[3]{a} \cdot \sqrt[3]{b} \cdot \sqrt[3]{c}.$$

$$a + b + c = 3 \sqrt[3]{abc}$$

CBS

$$(a + b + c)^3 = 27abc.$$

QUESTION



If $\frac{(a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3} = m(a+b)(b+c)(c+a)$, let E

where a, b, c are distinct real numbers then m is equal to.

$$a^2 - b^2 = A$$

$$b^2 - c^2 = B$$

$$c^2 - a^2 = C$$

$$0 = A + B + C$$

$$A^3 + B^3 + C^3 = 3ABC$$

$$a - b = p$$

$$b - c = q$$

$$c - a = r$$

$$0 = p + q + r$$

$$p^3 + q^3 + r^3 = 3pqr$$

$$E = \frac{A^3 + B^3 + C^3}{p^3 + q^3 + r^3}$$

$$E = \frac{3ABC}{3pqr} = \frac{(a^2-b^2)(b^2-c^2)(c^2-a^2)}{(a-b)(b-c)(c-a)}$$

$$E = (a+b)(b+c)(c+a)$$

$$m = 1$$

$$\sqrt{21 - 4\sqrt{5} + 8\sqrt{3} - 4\sqrt{15}} = \sqrt{2^2 + (2\sqrt{3})^2 + \sqrt{5}^2 - 2 \cdot 2 \cdot \sqrt{5} + 2 \cdot 2 \cdot 2\sqrt{3} - 2 \cdot 2 \cdot \sqrt{3} \cdot \sqrt{5}}$$

A $\sqrt{5} - 2 + 2\sqrt{3}$ $= \sqrt{(2 + 2\sqrt{3} - \sqrt{5})^2} = |2 + 2\sqrt{3} - \sqrt{5}|$

$$= 2 + 2\sqrt{3} - \sqrt{5}$$

$$= 2 + \sqrt{12} - \sqrt{5}$$

$$= \sqrt{4} + \sqrt{12} - \sqrt{5}$$

B $-\sqrt{5} - \sqrt{4} - \sqrt{12}$

~~**C**~~ $-\sqrt{5} + \sqrt{4} + \sqrt{12}$

D $-\sqrt{5} - \sqrt{4} + \sqrt{12}$

QUESTION



Find the square root of $10 + \sqrt{24} + \sqrt{60} + \sqrt{40}$.

Tah04

QUESTION



Tan05

If $a_1 + a_2 + a_3 + a_4 = -3$ and $a_1^2 + a_2^2 + a_3^2 + a_4^2 = 63$ then find value of :

$$a_1a_2 + a_2a_3 + a_1a_3 + a_3a_4 + a_1a_4 + a_2a_4 = ?$$

(where $a_1, a_2, a_3, a_4 \in \mathbb{R}$)

$$* a^m \cdot a^n = a^{m+n}$$

$$* \frac{a^m}{a^n} = a^{m-n}$$

$$* (a^m)^n = a^{mn}$$

$$* a^0 = 1 \quad (a \neq 0) \rightarrow \frac{a^m}{a^m} = 1 \quad (a \neq 0)$$

$$a^{m-m} = 1$$

$$a^0 = 1$$

$$* a^{-m} = \frac{1}{a^m} \quad (a \neq 0)$$

$$\searrow a^m \cdot a^{-m} = a^{m+(-m)} = a^0 = 1$$

$$a^{-m} = \frac{1}{a^m}$$

$$* (a^m b^n)^p = a^{mp} \cdot b^{np}$$

$$* a^{m/n} = (a^m)^{1/n} = (a^{1/n})^m$$

$$* a^{1/n} = \sqrt[n]{a} \quad n \in \mathbb{N}, n \geq 2$$



Laws of Exponents



$$0^m = 0, m > 0$$
$$0^0, 0^{-m} \text{ is Not def.}$$

Name of Exponent Rules	Rule
Zero Exponent Rule	$a^0 = 1$ (Where $a \neq 0$)
Identity Exponent Rule	$a^1 = a$
Product Rule	$a^m \times a^n = a^{m+n}$
Quotient Rule	$a^m / a^n = a^{m-n}$
Negative Exponents Rule	$a^{-m} = 1/a^m; (a/b)^{-m} = (b/a)^m$
Power of a Power Rule	$(a^m)^n = a^{mn}$
Power of a Product Rule	$(ab)^m = a^m b^m, (a^p b^q)^\alpha = a^{p\alpha} b^{q\alpha}$
Power of a Quotient Rule	$(a/b)^m = a^m / b^m$
Fractional Rule	$a^{1/n} = \sqrt[n]{a}; a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m} = (a^{1/n})^m = (\sqrt[n]{a})^m$

$$\star a^x = 1 \begin{cases} x=0, a \neq 0 \\ a=1, x \in \mathbb{R} \\ a=-1 \text{ \& } a^x = 1 \end{cases}$$

Ex: $(-1)^4 = 1$

$$(-1)^{\frac{4}{3}} = \left((-1)^{\frac{1}{3}}\right)^4 = (-1)^4 = 1.$$

$$\star a^x = a^y \begin{cases} x=y \\ a=1 \\ a=-1 \text{ \& } (-1)^x = (-1)^y \\ a=0 \text{ \& } x, y > 0. \end{cases}$$

$$(-1)^7 = (-1)^9$$

$$(-1)^{10} = (-1)^2$$

QUESTION

$$(1^3 + 2^3 + 3^3 + 4^3)^{-3/2} =$$

$$\left(\left(\frac{4(4+1)}{2} \right)^2 \right)^{-\frac{3}{2}}$$

$$(10^2)^{-\frac{3}{2}}$$

$$= 10^{-3}$$

~~A~~

10^{-3}

B

10^{-2}

C

10^{-4}

D

10^{-1}

$$\star \sum_{r=1}^n r = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$\star \sum_{r=1}^n r^2 = 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\star \sum_{r=1}^n r^3 = 1^3 + 2^3 + \dots + n^3 = \left(\frac{n(n+1)}{2} \right)^2$$

$$\star \sum_{r=1}^n (2r-1) = 1 + 3 + 5 + 7 + \dots + (2n-1) = n^2$$

$$\star \sum_{r=1}^n (2r) = 2 + 4 + 6 + \dots + 2n = n(n+1)$$

QUESTION

Tah06

If $\sqrt[4]{\sqrt[3]{x^2}} = x^k$, then $k =$

A $\frac{2}{6}$

B 6

C $\frac{1}{6}$

D 7

QUESTION



Tan07

The numerical value of $(x^{1/a-b})^{1/a-c} \times (x^{1/b-c})^{1/b-a} \times (x^{1/c-a})^{1/c-b}$ is
(a, b, c are distinct real numbers)

- A** 1
- B** 8
- C** 0
- D** None

QUESTION



$$\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots \infty \text{ times}}}}} = x$$

$$\sqrt{6 + x} = x$$

$$x^2 = x + 6$$

$$x^2 - x - 6 = 0$$

$$x = 3, -2$$

~~A~~ 3

B 2

C 1

D ± 3

QUESTION



$\sqrt{5 + \sqrt{5 + \sqrt{5} + \dots \infty}}$ is equal to

Tah08

A 5

B $5 + \sqrt{5}$

C $\frac{1 + \sqrt{21}}{2}$

D $\frac{\sqrt{5} - 1}{2}$

QUESTION



Tah09

If $a, b, c \in \mathbb{R}$ and $a, b, c \neq 0$ such that $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 6$ and $\frac{b}{a} + \frac{c}{b} + \frac{a}{c} = 8$, then $\frac{a^3}{b^3} + \frac{b^3}{c^3} + \frac{c^3}{a^3} - 3$ is equal to

- A** 81
- B** 48
- C** 72
- D** 84

QUESTION



Tan 10

Let $a, b, c \in \mathbb{N}$ ($a > b$) satisfy $c^2 - a^2 - b^2 = 101$ with $ab = 72$. Then which of the following can be correct?

- A** b and c are coprime
- B** c is an odd prime
- C** $(a + b + c)$ is even
- D** $a + b = c + 1$

Ans. A, D

**Saari Class Illustrations
Retry karni Hai**



Today's KTK



No Selection $\xrightarrow{\text{TRISHUL Apnao IIT Jao}}$ **Selection with Good Rank**





If a , b , & c are three non zero real numbers such that $5a^2 + 4b^2 + 4c^2 - 8ab - 4ac = 0$ then the value of $a/b + b/c$ is _____



If a , b , & c are three non zero real numbers such that $2a^2 + b^2 + c^2 - 2ab - 2ac = 0$ then the value of $\frac{a+b}{c}$ is equal to _____



If $x^2 + 16y^2 + 9z^2 = 4xy + 12yz + 3zx$ then find the value of $\frac{x+4y}{3z}$. (Given $x, y, z \in \mathbb{R}_0$)



If the real numbers x, y, z are such that $x^2 + 4y^2 + 16z^2 = 48$ and $xy + 4yz + 2zx = 24$, what is the value of $x^2 + y^2 + z^2$?



If x, y, z are real numbers then find the minimum value of $4x^2 + y^2 + 9z^2 - 4x - 2y - 6z + 17$.

Solution to Previous TAH

QUESTION



If $81x^5 + 27x^3 - 9x^2 + 50$ is divided by $(3x + 2)$ then remainder is λ then

- A** $\frac{3\lambda - 42}{10}$ is equal to 4
- B** if $\lambda = \frac{p}{q}$ then $(p + q)$ is divisible by 17 (where p & q are coprime)
- C** λ is a natural number
- D** $\left(\lambda - \frac{1}{3}\right)$ is divisible by 3

Ans. A, B, D

36) If $81x^5 + 27x^3 - 9x^2 + 50$ is divided by $(3x+2)$ then remainder is λ then -

$$P(x) = 81x^5 + 27x^3 - 9x^2 + 50$$

$$\therefore P(-2/3) = \lambda$$

$$\Rightarrow 81 \times \left(-\frac{2}{3}\right)^5 + 27 \left(-\frac{2}{3}\right)^3 - 9 \left(-\frac{2}{3}\right)^2 + 50 = \lambda$$

$$\Rightarrow -\frac{32}{3} - 8 - 4 + 50 = \lambda$$

$$\Rightarrow 50 - \frac{32}{3} - 12 = \lambda$$

$$\Rightarrow \frac{150 - 32 - 36}{3} = \lambda$$

$$\Rightarrow \lambda = \frac{150 - 68}{3} = \frac{82}{3}$$

$$\frac{3\lambda - 42}{10} = \frac{82 - 42}{10} = \frac{40}{10} = 4$$

$$82 + 3 = 85 = 5 \times 17$$

$$\frac{82}{3} - \frac{1}{3} = \frac{81}{3} = 27 = 3 \times 9$$

\therefore Ans \rightarrow (a), (b), (d)

QUESTION



Let $f(x) = x^{135} + x^{125} - x^{115} + x^5 + 1$. If $f(x)$ is divided by $x^3 - x$ then the remainder is some function of x say $g(x)$. Find the value of $g(10)$.

37) Let $f(x) = x^{135} + x^{125} - x^{115} + x^5 + 1$. If $f(x)$ is divided by $x^3 - x$ then the remainder is some f^n of x say $g(x)$. $g(10) = ?$

$$f(x) = x^{135} + x^{125} - x^{115} + x^5 + 1$$

$$(x^3 - x) = x(x+1)(x-1)$$

$$\therefore f(0) = 0 + 0 - 0 + 0 + 1 = 1$$

$$f(-1) = -1 - 1 + 1 - 1 + 1 = (-1)$$

$$f(1) = 1 + 1 - 1 + 1 + 1 = 3$$

$$f(x) = (x^3 - x)Q(x) + g(x)$$

$$\text{Let } g(x) = ax^2 + bx + c$$

$$f(x) = (x^3 - x)Q(x) + (ax^2 + bx + c)$$

$$f(1) = a + b + c = 3$$

$$f(0) = c = 1$$

$$f(-1) = a - b + c = (-1)$$

$$\therefore \begin{array}{l|l} c=1, a+b+c=3 & a-b+c=(-1) \\ \hline \therefore a+b=2 & \therefore a-b=(-2) \end{array}$$

$$\therefore \begin{array}{l|l} 2a=0 & b=2 \\ \hline a=0 & \end{array}$$

$$\begin{aligned} \therefore g(x) &= ax^2 + bx + c \\ &= 0 + 2x + 1 \\ &= 2x + 1 \end{aligned}$$

$$g(10) = 20 + 1 = 21$$

(Ans)

QUESTION



If $p(x)$ is a polynomial of 3 degree for which $p(1) = 1$, $p(2) = 4$, $p(3) = 9$ then find the value of $p(5)$ and leading coefficient be 2.

Q8) If $p(x)$ is a polynomial of 3 degree for which $p(1)=1$, $p(2)=4$, $p(3)=9$ then find the value of $p(5)$ and leading coefficient be 2.

$$g(x) = p(x) - x^2$$

$$g(1) = p(1) - 1 = 1 - 1 = 0$$

$$g(2) = p(2) - 4 = 4 - 4 = 0$$

$$g(3) = p(3) - 9 = 9 - 9 = 0$$

$$g(x) \text{ roots factors} = (x-1), (x-2), (x-3)$$

$$\therefore g(x) = a(x-1)(x-2)(x-3)$$

$$\Rightarrow p(x) - x^2 = a(x-1)(x-2)(x-3)$$

$$\therefore p(x) = 2(x-1)(x-2)(x-3) + x^2$$

$$p(5) = 2 \times 4 \times 3 \times 2 + 25$$

$$= 48 + 25$$

$$= 73$$

$$\therefore p(5) = 73 \text{ (Ans)}$$



Mann Ki Baat Ashish Sir ke Saath

SHORT NOTES

❖ Class Notes Bananaa → MY HAND WRITTEN NOTES

Rough copy + Notes copy — Theoretical points.

❖ Question Liknaa → Blank PPT ✓

Tum Bs Question Practice Karo ✓

THANK
YOU